

Momentum in Tennis: A Modeling Study

Summary

Research Purpose: The purpose of this study is to develop a mathematical model that analyzes the flow of tennis matches, identifies dominant players, and predicts game outcomes. The study aims to explore the concept of momentum in tennis and its impact on match results.

Content Overview: The study begins with an introduction, providing background knowledge on momentum in sports and its relevance in tennis. It then introduces the dataset used in the study, which includes comprehensive information on Gentlemen's singles matches at Wimbledon 2023. model assessment, swings prediction, and experimental analysis.

Detailed Explanation: The study applies statistical analysis and machine learning techniques to develop a model that evaluates player performance and quantifies momentum. It utilizes the provided dataset to identify factors affecting momentum and assess their impact. The study also includes a model assessment to determine the correlation between momentum and game results, using statistical analysis and random simulations. Additionally, a predictive model is developed to forecast momentum fluctuations and turning points in tennis matches. The developed model is then evaluated using data from other tennis matches and a different sports event, measuring its accuracy in predicting swings and momentum shifts.

Real-world Significance: The research contributes to a better understanding of momentum in tennis matches, providing valuable insights for coaches, players, and tennis enthusiasts. The developed models and analysis techniques can aid in performance evaluation, match prediction, and strategy development in the sport.

Keywords: Tennis; Momentum; Model Assessment

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1 Introduction

1.1 Background Knowledge

In many sports fields, the analysis and prediction of game results have been paid much attention, and the related concept of momentum has been proposed and discussed. In physics, momentum represents the intensity or force gained by an object or series of events in motion [1]. When used in sports, such as tennis, momentum refers to the strength or advantage a player or team gains during a game. The highly anticipated showdown between Carlos Alcaraz and Novak Djokovic in the 2023 Wimbledon men's singles final displayed not only extraordinary skill, but also a distinct wave of momentum that has raised concerns about the nature and impact of the phenomenon. To understand "momentum" in tennis requires recognizing the shifting balance of a match, the ups and downs of a player's performance, and the impact these factors have on the overall outcome of a match. The technical statistics of the winners and losers of the game often show the advantages and disadvantages of each other. What technical indicators and external factors play a decisive role in the outcome of professional tennis matches is worth exploring [2]. Therefore, the analysis of real-time match data (such as serve reception, break point, etc.) can effectively predict the trend and result of tennis matches.

1.2 An issue to be solved

Numerous researchers have engaged in diverse methodologies to conceptualize a tennis match, employing probabilistic modeling or leveraging machine learning frameworks to predict match results [3]. The goal of our team is to build a mathematical model that analyses the flow of the game, identify the dominant players, and predict whether a game will be won or lost. To fulfill these objectives, our team utilized the data provided for the Wimbledon 2023 men's matches after the first 2 rounds, focusing on the aforementioned Gentlemen's final for detailed analysis. Rigorous statistical approach has been used to develop, validate, and test the model on different matches, which gauges the performance and generalizability of the model across various match conditions.

1.3 Introduction to the dataset

The dataset used in this study encompasses a comprehensive set of information pertaining to the Gentlemen's singles matches held at Wimbledon in 2023, specifically focusing on matches occurring after the second round. This dataset plays a pivotal role in facilitating a comprehensive understanding of the dynamics of player performance, capturing the intricate and continuous progression of matches, and enabling analysis of the decisive moments that contribute to victory or defeat. The dataset incorporates crucial match-specific details, including scoring information, game outcomes, set results, tie-breaker statistics, serving metrics, rest breaks, and other pertinent match characteristics. With its wealth of information, this dataset serves as the cornerstone of our research, providing invaluable insights into the realm of tennis match dynamics and performance evaluation.

By leveraging this dataset, researchers gain access to a rich source of data that enables them to delve into various aspects of player performance and match dynamics. The dataset empowers researchers to investigate the influence of different variables on match outcomes, such as the

impact of serving statistics on winning probabilities or the correlation between rest breaks and player performance. Additionally, researchers can explore the temporal dynamics of matches, examining how the momentum shifts during critical moments of the game and identifying patterns that contribute to success or failure. Furthermore, the dataset allows for the analysis of long-term performance trends, enabling researchers to assess the consistency of players' performance over multiple matches and uncover factors that contribute to sustained success.

This dataset fosters a rigorous and data-driven approach to studying tennis match dynamics, offering researchers the opportunity to employ various statistical and analytical techniques. By applying methods such as regression analysis, correlation analysis, and data visualization, researchers can extract meaningful insights from the dataset and make evidence-based conclusions. The comprehensive nature of the dataset ensures that researchers can explore a wide range of research questions, thereby advancing our understanding of the intricacies of tennis matches and providing valuable insights for coaches, players, and tennis enthusiasts.

In conclusion, the dataset comprising information on the Gentlemen's singles matches at Wimbledon 2023 serves as a fundamental resource for investigating player performance dynamics, analyzing match progression, and evaluating key moments leading to victory or defeat. Its extensive coverage of match-specific details empowers researchers to delve into various aspects of tennis match dynamics and enhances our understanding of the sport.

2 Analysis of the Problem

2.1 Problem: Player Performing Model

In this study, we aim to develop a model that captures the game process in tennis matches and evaluates the performance of each player at any given time. The goal is to determine which player performs better and quantify their level of performance advantage, which is referred to as momentum in this article.

We will first process the original data set, obtain the factors that affect the momentum based on the given data, and explore the impact of each factor on the momentum, and quantify the value through a weighted method.

2.2 Problem: Model Assessment

In this study, we need to evaluate a tennis coach's statement that momentum in a tennis match is simply the result of random fluctuations and that fluctuations in a match have no significant impact on a player's success. To address this issue, we propose the following method using statistical correlation analysis and randomness testing: According to statistical analysis, we will get the correlation coefficient, through which we can see whether there is a strong correlation between momentum and game results.

In addition to this, in order to more intuitively compare the views of this tennis coach, we will perform a random simulation of a tennis match in which winning points are completely randomly generated. By simulating the game multiple times, we can obtain the distribution of momentum scores under these random conditions. Next, we compare the distribution of momentum scores obtained from actual race data with the results of random simulations. If the momentum scores

in the actual game deviate significantly from the results of the random simulation, it is a sign that the momentum in the game is not just driven by random fluctuations.

By following this approach, we can objectively evaluate the coach's claims and determine whether momentum plays a significant role in a tennis match or if it just happens randomly. The results of this analysis will provide valuable insights into the nature of momentum in tennis.

2.3 Problem: Swings Prediction

The current task involves developing a model that can predict fluctuations in the flow of a tennis match, indicating when the momentum will shift from favoring one player to the other. We aim to forecast the turning points of momentum based on the developed momentum model.

To address this problem, we will utilize the provided match data and apply established mathematical modeling, machine learning, and statistical analysis techniques. In particular, we will employ a cross neural network to discern momentum transitions.

By following this approach, we can develop a predictive model that identifies momentum fluctuations in a tennis match and determines the factors most related to these fluctuations. The insights gained from this analysis can aid coaches and players in comprehending and anticipating changes in momentum, thereby providing a competitive advantage in future matches.

2.4 Problem: Experimental Analysis

In this study, we aim to evaluate the performance of the developed model by applying it to other tennis matches and an additional sports event. Specifically, we will assess the model's ability to predict swings in the match and quantify the momentum at each moment. Furthermore, we will investigate how well the model performs across different match types, tournaments, court surfaces, and even in a different sport such as table tennis.

To conduct this evaluation, we utilized data from two major events: the 2022 US Open (men's singles matches) and the 2022 BWF World Badminton Championships (singles matches). Using our developed model, we applied it to the selected matches, quantifying the momentum at each moment and predicting the winning rate based on the momentum scores. By comparing these predictions with the actual match outcomes, we can assess the model's accuracy in capturing the swings and momentum shifts during the games.

3 Player Performing Model

3.1 Preprocessing

We performed the following preprocessing steps on the dataset *Wimbledon_featured_matches.csv*.

(1) Outlier handling: For several relatively continuous variables, namely *p1_distance_run*, *p2_distance_run*, and *rally_count*, we dealt with outliers.

To identify outliers and determine their upper and lower limits, we utilized a box plot analysis. Figure 1 displays the generated box plot. In the plot, the box represents the interquartile range of the distance run, the median is indicated by the central line, and the whiskers depict the overall range of the data. Outliers are represented by individual data points beyond the whiskers. Upon

observation, we noticed a significant number of outliers arising from a substantial variation in the distance run specifically when the rally count was 1. In order to ensure the accuracy and reliability of the data, we made the decision to handle these outliers by excluding the instances in which the rally count was 1. By excluding these outliers, we can obtain more precise results in subsequent analyses. This approach allows for a more accurate understanding of the underlying patterns and characteristics of the data.

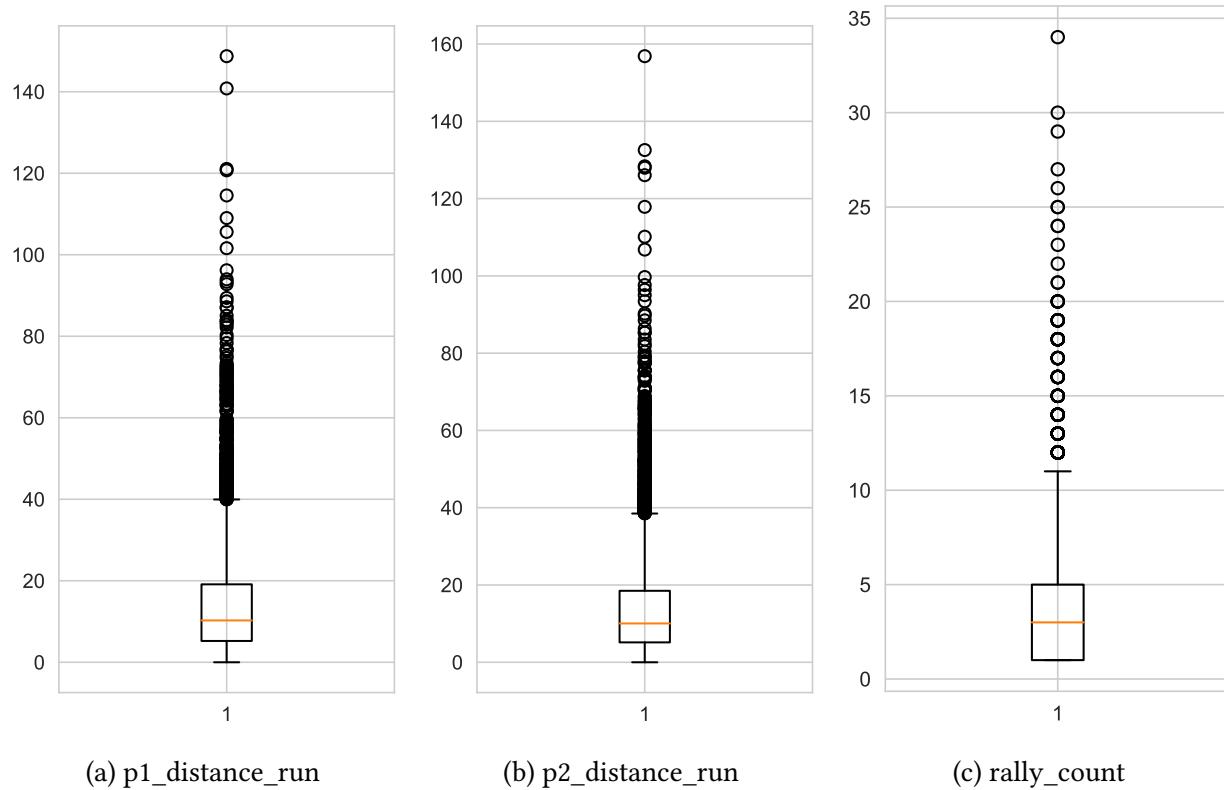


Figure 1: Boxplot based on the data

(2) Missing data handling: We discovered multiple records with a value of 0 in the *rally_count* data, specifically for match 1310, where the number of strokes per set was not recorded. By examining the data from other matches, we observed a positive correlation between *rally_count* and the players' running distance. Hence, we established a mapping relationship between running distance and stroke count to impute the missing values based on their correlation.

(3) Data standardization: We scaled the data to a zero mean and unit variance interval to ensure non-dimensionalization of multiple features. The collected data in the appendix was relatively complete, with only a few matches having a number of sets exceeding 24. There were no excessive outliers, so we did not perform extensive processing. However, for match 1303, we observed that the minutes part was continuous, suggesting an error in recording the hour. Therefore, we subtracted twenty-four from the time to obtain reasonable time data.

Figure 2: Examples of missing data, dirty data, and data recording error.

3.2 Feature Engineering

According to the common rules of tennis matches and player analysis, we have selected model indicators. Based on the review by Kilit *et al.* [4], we learned that certain indicators are crucial for the outcome of tennis matches, such as distance covered, rally count, and breaking points. Furthermore, considering the results of data set filtering and feature engineering, along with the correlation of the following indicators with winning probability, we have selected the following data as our model input:

point_victor, p1_distance_run, p2_distance_run, rally_count, p1_break_pt_won, p2_break_pt_won, p1_double_fault, p2_double_fault, p1_unf_err, p2_unf_err.

By processing the above data, we obtained five main indicators, which are as follows:

1. Point Victor: This indicator reflects which side is currently in an advantageous position. Based on the definition of momentum as "strength or force gained by motion or by a series of events," we intuitively observe that the point victor usually has more drive and confidence. The point victor tends to maintain their advantage, while the point loser may strive to turn the tide.
2. Difference in Distance Covered between Players: This indicator quantifies the level of activity of the players on the court. The distance covered between players within a set greatly impacts their physical stamina. If a player is frequently moved during rallies, resulting in covering a greater distance than the opponent, it can decrease the overall winning probability and diminish momentum.
3. Breaking Point: Breaking points are crucial moments because if the point loser loses the

point, the point victor gains an advantage in the game. Therefore, the occurrence of breaking points can have a significant impact on momentum and can be considered a critical turning point.

4. Forced and Double Fault Errors: Errors are important factors in a match. Forced errors indicate a player making a mistake due to the opponent's excellent performance, while double fault errors occur when a player commits a fault during serving. Both of these situations can change the course of the match, as they may provide opportunities for the point loser.

5. Consecutive Winning Points: Consecutive winning points refer to a player winning multiple points in a row. This can be seen as a continuation of momentum, as a player may continuously win points over a period of time, enhancing their confidence and putting pressure on the point loser.

Regarding the scoring factors, due to the inherent 'serving advantage' in tennis, we need to exclude this advantage from the quantification of momentum to obtain a fair assessment. Based on the current dataset and the analysis of the confusion matrix using the 'server' and 'point_victor' columns, we found that the server has a 67.3% probability of winning. Figure 3 (a) displays the generated confusion matrix. We restore it to an initial equal winning probability, resulting in a server coefficient of $\gamma = 74.2\%$. On a point won by the server, we multiply the score by the server coefficient γ to obtain a fair score indicator.

For the difference in distance covered factor, as mentioned in the data processing, the distance covered by players has a significant correlation with rally count. Therefore, we calculate the average difference in distance covered for each point to quantitatively assess this score indicator fairly.

By integrating and analyzing all the data, based on the confusion matrix analysis, it can be considered that there is a 58.9% probability that the player with a greater difference in distance covered has a lower winning rate. Figure 3 (b) displays the generated confusion matrix.

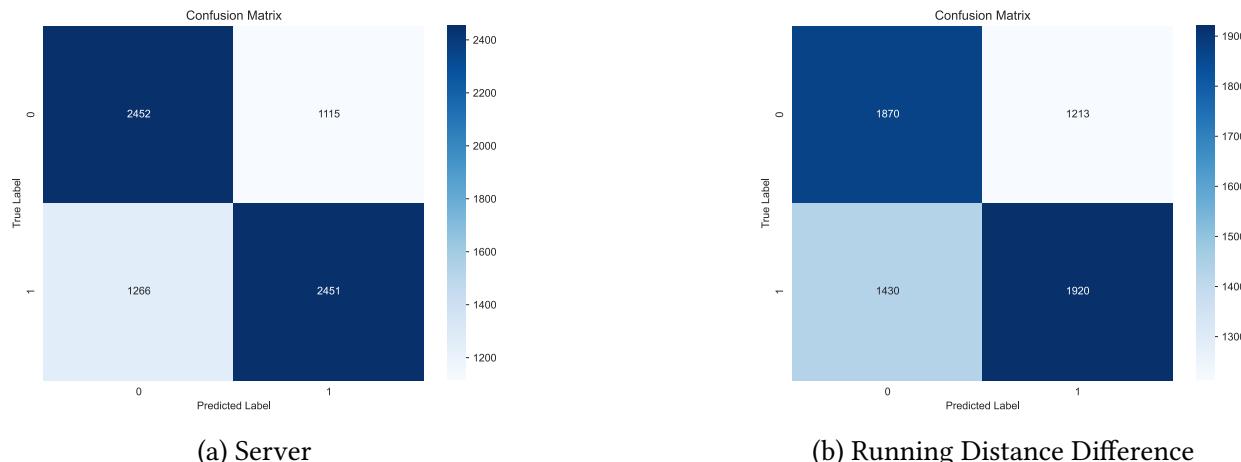


Figure 3: Confusion matrix is a commonly used performance evaluation tool for classification models, used to visualize the difference between the predicted results of the classification model and the actual labels. This is used to evaluate the correlation between our conjecture and the actual results

For the breaking point factor, we consider whether a point results in a break of serve as a scoring indicator.

For the error factor, we consider whether a point involves a forced error or a double fault as a scoring indicator. It is certain that it is not possible for both errors to occur simultaneously during a point.

For the consecutive winning points factor, since the impact of consecutive wins differs for each player, we quantify this score indicator as the cumulative value of consecutive wins. Starting from the point won, for each consecutive win, we add 0.2 to the score. For example, if a player wins two consecutive points, their score for this indicator would be 0.2.

It is worth noting that all our indicators have both positive and negative signs. If an indicator is positive, it signifies an advantage for Player 1, and vice versa.

The five scoring indicators we have derived in figure 4, along with their calculation formulas.

Indicator	Formula	Explanation
<i>score</i>	$\begin{cases} \pm\gamma, & \text{scoring by the serve} \\ \pm 1, & \text{otherwise} \end{cases}$	If Player 1 scores, the indicator takes a positive value; otherwise, it takes a negative value.
<i>run</i>	$\frac{d_1 - d_2}{\text{rallycount} - 1}$	d_1 is the distance covered by Player 1, d_2 is the distance covered by Player 2.
<i>break</i>	$\begin{cases} \pm 1, & \text{scoring at breaking point} \\ 0, & \text{otherwise} \end{cases}$	If Player 1 scores at a breaking point, the indicator takes a positive value; otherwise, it takes a negative value.
<i>miss</i>	$\begin{cases} \pm 1, & \text{error occur} \\ 0, & \text{otherwise} \end{cases}$	If Player 1 makes an error, the indicator takes a negative value; otherwise, it takes a positive value.
<i>cont</i>	$0.2(c - 1)$	c is the number of consecutive winning points. This parameter has a positive or negative value depending on whether Player 1 has a winning streak or not.

Figure 4: The name of figure

3.3 Proposed Model

3.3.1 Quantification of Metrics

Considering the complexity of tennis match scores and the difficulty of visualizing the data in an intuitive manner, which makes it challenging to validate our results, we propose a method for quantifying metrics to facilitate visualization analysis in conjunction with our proposed model.

Our metric quantification considers two types of data in tennis: game victories (game scoring) and point victories (pointing). This utilizes two metrics from the dataset:

$p1_games_won$, $p2_games_won$, and $p1_points_won$, $p2_points_won$. These two metrics provide a macroscopic and microscopic view of the current situation. In a tennis match, although there are many complex rules, game victories are crucial for overall success. Therefore, the sum of game victories represents the macroscopic situation. Pointing reflects the immediate performance of the players, so the difference in points can to some extent represent the current direction of the match.

For these two metrics, we propose:

$$\begin{aligned} \text{game_difference} &= p1_games_won - p2_games_won \\ \text{point_difference} &= p1_points_won - p2_points_won \end{aligned}$$

We used two representative examples, 1301 and 1304, as references to visualize the metric method proposed above. This metric evaluates the competition based on the most basic scores.

According to Figure 5, it can be seen that in fact, the two indicators have a certain degree of judgment ability for the changes in the situation on the field. In match 1301, the pointing was basically consistent with the game scoring. However, it can be seen in match 1304 that there is a significant deviation between pointing and game scoring. This is because in the 30min-1h race, one side had an overwhelming pointing advantage, but there was not much difference in game scoring. Even if the game scoring is pulled back afterwards, there will still be differences in pointing. This example typically illustrates why we need two metrics.

3.3.2 Model Architecture

Our model aims to capture the momentum of each player's performance on the court using five metrics and a persistence term.

We establish the momentum model as follows: Momentum, denoted as m , is modeled using a persistence term that captures the accumulation effect over time. Silva *et al.* [5] mentioned in their study on psychological momentum in tennis that psychological momentum has a certain level of persistence in tennis matches. Therefore, by incorporating persistence, we can introduce the concept of persistent psychological momentum into our momentum model for player performance.

During preprocessing, time t is transformed into an integer variable to establish the model representing each point. m_t represents the momentum at time t , m_{t-1} represents the momentum at time $t - 1$, ρ represents the persistence coefficient, and Δm represents the momentum change at the current time step, which is calculated using the following five metrics.

$$m_t = \rho \cdot m_{t-1} + \Delta m \quad (1)$$

This equation indicates that the momentum at the current time step is a fraction of the previous momentum plus the current momentum change.

This formulation can simulate the effect of persistent momentum. The persistence coefficient ρ determines the contribution of the previous momentum to the current momentum. If ρ is close to 1, the influence of the previous momentum on the current time step will be significant, resulting

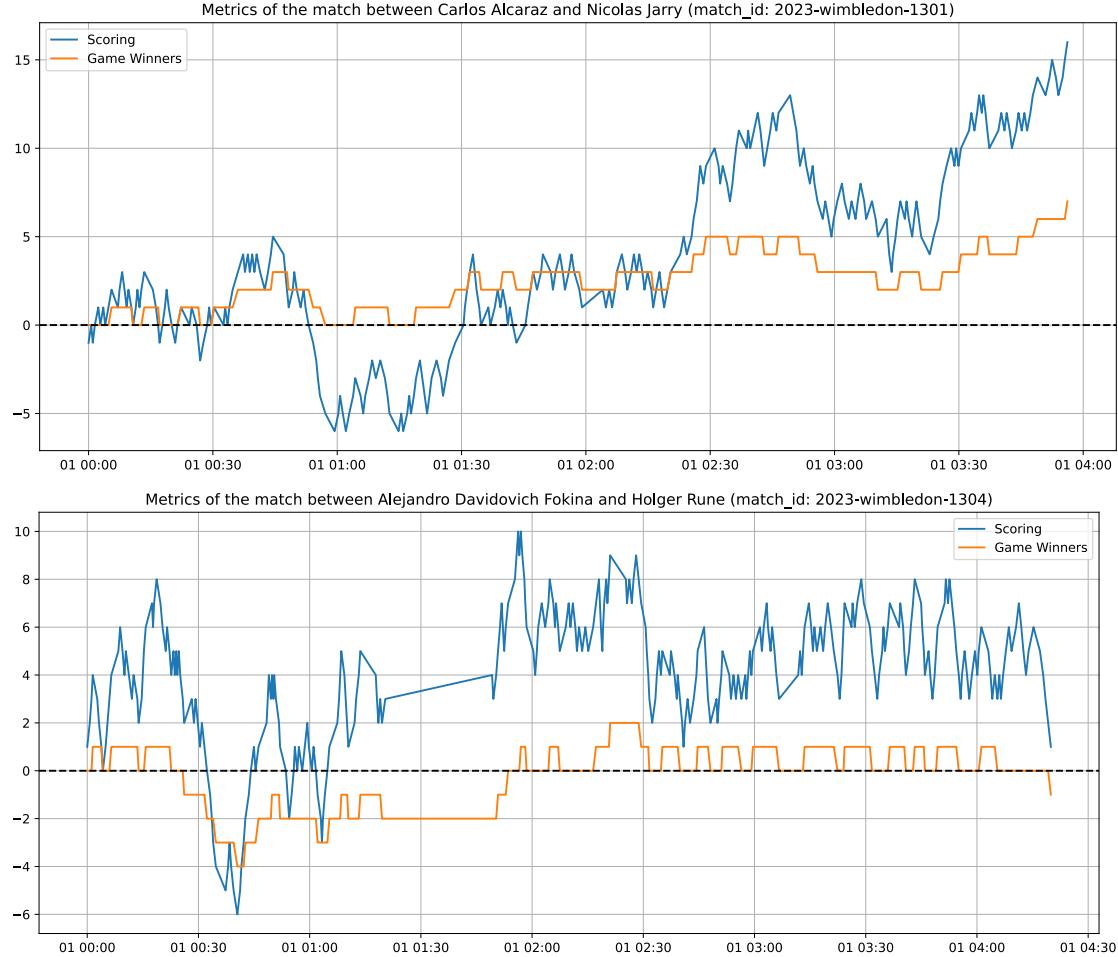


Figure 5: Visualization of pointing and game scoring in match 1301 and 1304.

in a more persistent momentum. Conversely, if ρ is close to 0, the influence of the previous momentum on the current time step will be smaller, causing the momentum to decay faster. This coefficient is related to the on-court situation and the psychological factors of different players.

Regarding Δm , it is calculated as follows. Through feature engineering, we propose five metrics: Whether the point is scored (multiplied by a serving coefficient) s_{score} , Running difference s_{run} , Whether a break occurs s_{break} , Whether a mistake is made s_{miss} , Whether there is a winning streak s_{cont} ,

These metrics are combined using linear weighting. We define a weight vector where each element represents the weight chosen for each metric:

$$\mathbf{p} = [p_1, p_2, p_3, p_4, p_5]$$

By performing element-wise product, which is depicted as o-dot in the equation, Δm is:

$$\Delta m = \mathbf{s} \odot \mathbf{p} \quad (2)$$

Combining Equations (1) and (2), we obtain the momentum calculation formula at time t :

$$m_t = \rho \cdot m_{t-1} + \mathbf{s} \odot \mathbf{p} \quad (3)$$

It is important to note that the introduction of the persistence term increases the complexity of the momentum model. The value of the persistence coefficient ρ needs to be determined. Additionally, due to the varying situations in different matches and the differences between players in terms of personality, psychological state, technical characteristics, etc., the weights for each parameter need to be adjusted. The weight adjustment process is dynamic.

3.4 Parameter Tuning

We employed the Particle Swarm Optimization (PSO) algorithm for parameter optimization in our study.

In this part, we present the application of the PSO algorithm. The objective of this work is to optimize the performance of our model by finding the best set of parameter values that maximize its accuracy in capturing the momentum of players' performance in tennis matches.

3.4.1 Algorithm

PSO is a population-based optimization algorithm inspired by the social behavior of bird flocking or fish schooling [6]. It iteratively searches for the optimal solution by adjusting the positions and velocities of a group of particles in the search space. In our case, each particle represents a potential set of parameter values for the model.

The PSO algorithm consists of the following steps:

- Initialization: Initialize a population of particles with random positions and velocities within the parameter space.
- Evaluation: Evaluate the fitness of each particle by applying the model with the corresponding parameter values and comparing the results with the actual momentum data.
- Update Personal Best: Update the personal best position and fitness for each particle based on its current fitness and position.
- Update Global Best: Update the global best position and fitness by selecting the particle with the highest fitness value among all particles.
- Velocity and Position Update: Update the velocities and positions of each particle based on its personal best, global best, and the inertia weight.
- Termination: Repeat steps 2-5 until a termination criterion is met (e.g., maximum number of iterations or desired fitness level reached).

By iteratively updating the positions and velocities of particles, the PSO algorithm explores the parameter space and converges towards the optimal set of parameter values that yield the best model performance.

The velocities are updated using the following equation:

$$V_i(t+1) = w \cdot V_i(t) + c_1 \cdot r_1 \cdot (pbest_i - X_i(t)) + c_2 \cdot r_2 \cdot (gbest - X_i(t)) \quad (4)$$

where r_1 and r_2 are random numbers between 0 and 1, w , c_1 , and c_2 are parameters of the PSO algorithm, $pbest_i$ is the position that gives the best fitness value ever explored by particle i , and $gbest$ is the position explored by all particles in the swarm.

In this algorithm, we used a mock accuracy calculation process to compute the accuracy, which is the negative of loss. At each moment of a match, our model was used to calculate the momentum. At time t :

$$a_t = \begin{cases} 1, & (m > 0 \wedge \text{winner} = 1) \vee (m < 0 \wedge \text{winner} = 2) \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

, where winner is the winner of the next game. By considering the winner of the next game instead of the winner of the next point, we can better assess the current player's state. The accuracy is calculated as follows:

$$acc = \frac{\sum a_t}{n_t} \times 100\% \quad (6)$$

where σ_t denotes the sum over all moments, a_t is the value at moment t , and n_t is the total number of points.

3.4.2 Results

We conducted experiments using the PSO algorithm to determine the optimal parameter values for our proposed model. The parameter space was defined based on the plausible range of values for each parameter. In this experiment, the parameter space was set as arbitrary real numbers between -1 and 1. We set the maximum number of iterations to 100 and the population size to 50. Parameter c_1 is 0.5, c_2 is 0.3, w is 0.9.

The fitness function used for evaluation was the accuracy of predicting the momentum values in terms of positive or negative and the actual match outcomes. A higher accuracy indicates better model performance.

After running the PSO algorithm, we obtained the optimal parameter values that maximize the accuracy of our model in capturing the momentum of players' performance in tennis matches.

3.4.3 Visualization

We visualize the results of momentum and combine them with metrics to present the overall flow of the competition and the performance of players at a specific moment.

There are several options available for visualization. For momentum, we can choose whether to smooth the data. If the momentum fluctuates too much during a particular match, we can apply smoothing by taking the five-point moving average (i.e., window size = 5) to the momentum values.

We have selected cases 1701 and 1309 for demonstration. Regarding metrics, we have two options to choose from.

- For case 1309, we present the original momentum values combined with pointing metrics.

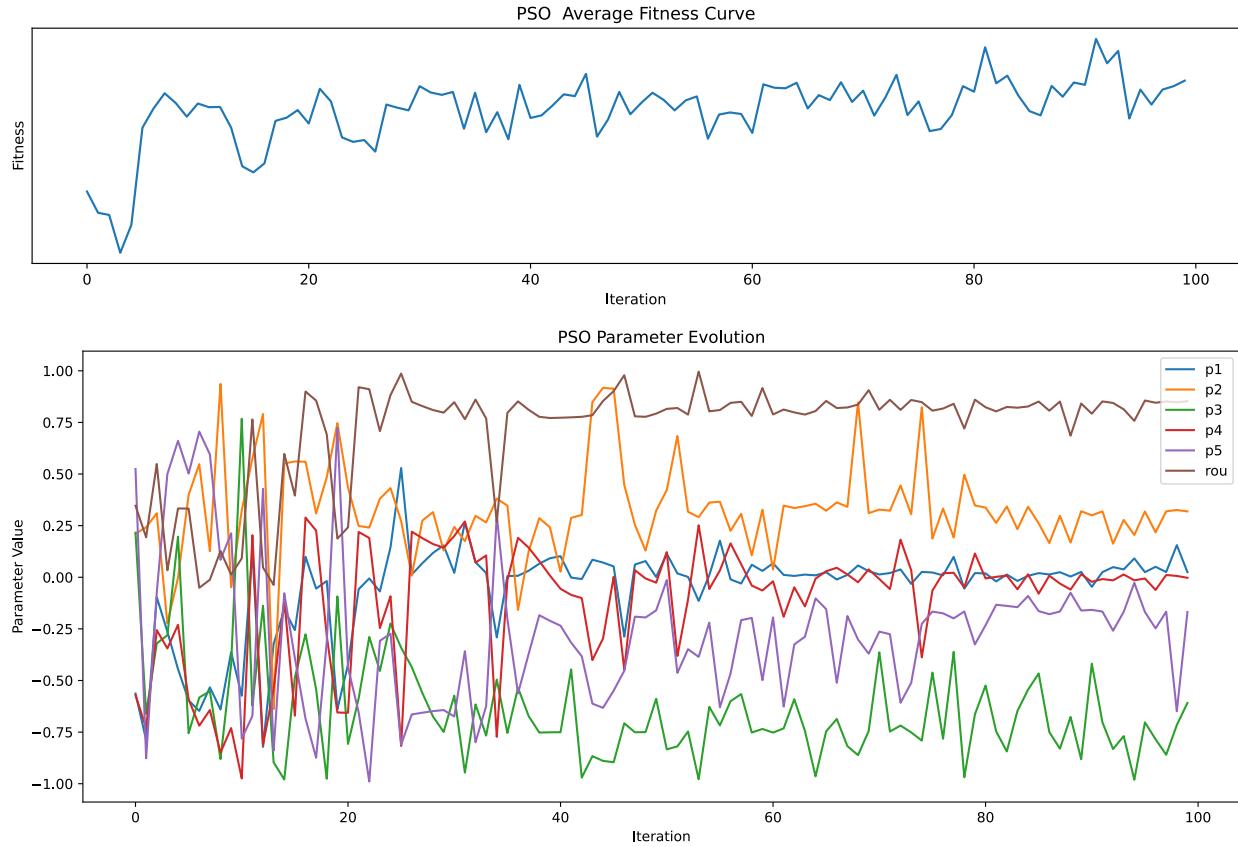


Figure 6: The fitness curve and parameter evolution plot of PSO process applying to match 1309. Although the convergence curve is an average with significant fluctuations, it can be seen from the graph that the parameters approach convergence at the end. The final accuracy is 76.3%.

- For case 1701, we present the smoothed momentum values combined with game scoring metrics.

In Figure 7, it is evident that both visualizations are highly intuitive.

We now provide an analysis and explanation for case 1701, as mentioned in the question.

The explanation for case 1701 is as follows: Djokovic seemed destined to win easily as he dominated the first set 6-1. However, the second set was tense and was finally won by Alcaraz in a tie-breaker 7-6. The third set was the reverse of the first, with Alcaraz winning handily 6-1. As the fourth set started, the young Spaniard seemed in total control, but the match once again changed course, with Djokovic taking complete control and winning the set 6-3. The fifth and final set began with Djokovic carrying the edge from the fourth set, but once again there was a change of direction, and Alcaraz gained control and ultimately won 6-4.

From the visualization, it can be observed that there was a significant positive-to-negative change in momentum from set 1 to set 2, which occurred at approximately 00:40. From then until 2:00, the momentum fluctuated around 0, indicating a closely contested match. Around 3:20, the momentum shifted towards player 2. Finally, it returned to player 1. These observations align

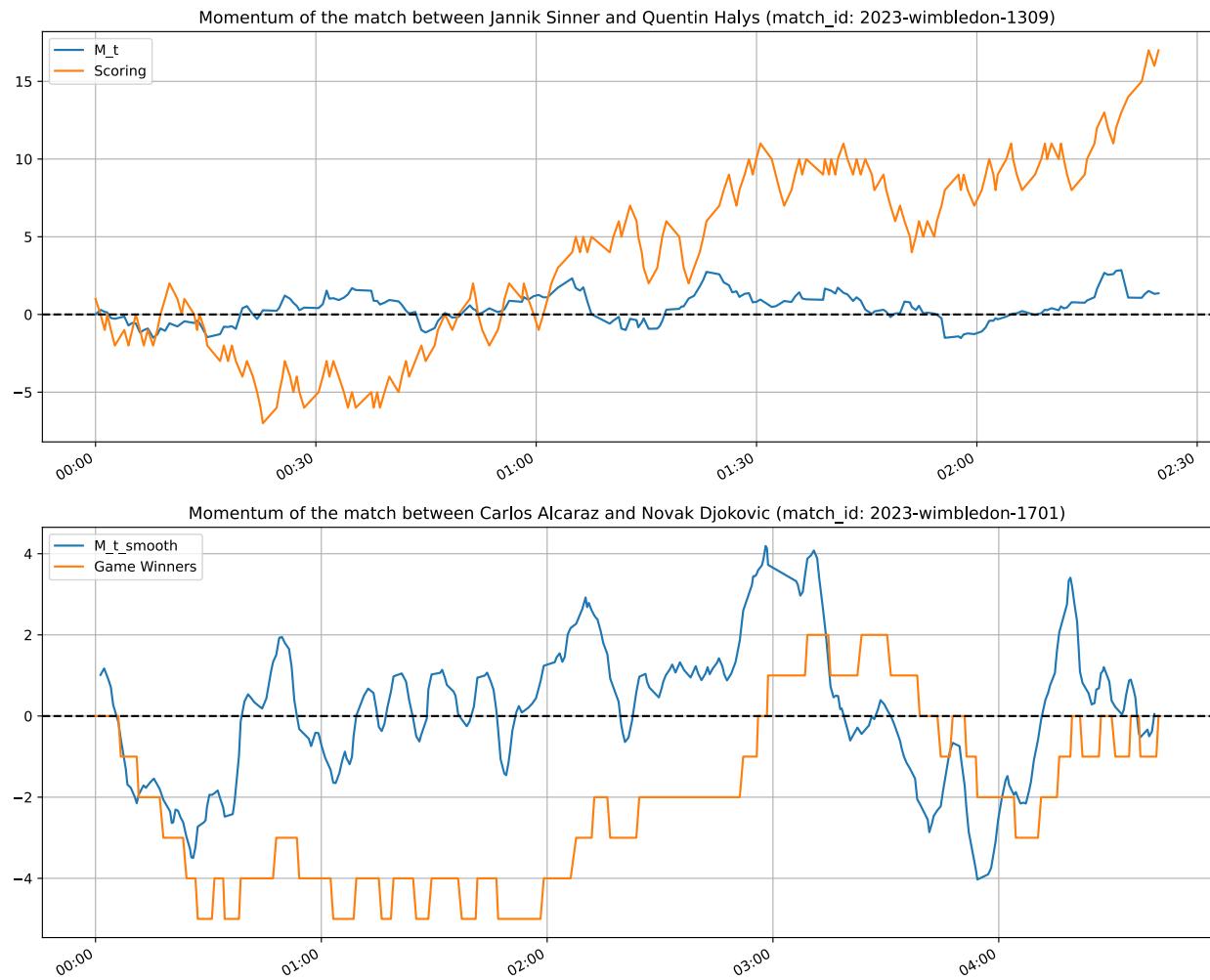


Figure 7: Visualization of momentum and metric in match 1701 and 1309.

well with our understanding of the match information and the game scoring metric.

4 Model Assessment

4.1 Statistic Analysis

First, we can assess the correlation between momentum scores and the outcome of the next game through statistical analysis.

If momentum indeed has a significant impact on game results, we should observe a positive correlation between high momentum scores and the player's winning percentage. We first evaluate the correlation between a player's momentum score and the outcome of the next game using the commonly used Spearman rank correlation coefficient, where the absolute value of the Spearman coefficient closer to 1 indicates a stronger correlation.

The formula for the Spearman correlation coefficient is:

$$\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

Based on the formula, we obtain $\rho = 0.7458435201718437$, indicating a strong correlation between the two variables within the range of 0.6-0.8, where $p_value = 4.312816162712459 \times 10^{-39}$.

Since in our model, "momentum" is quantified as a continuous variable, and "next game winner" is a binary variable, we need to seek a correlation analysis that is more suitable for the current situation. The point-biserial correlation is used in such specific cases when you have one binary variable and one continuous variable and want to measure the strength of the relationship between the two variables.

We denote player 1's score as 0 and player 2's score as 1. We want to explore the correlation between which player has stronger momentum and the next game winner. According to whether the next game winner is player 1 or player 2, we can divide the momentum data into two groups: group 1 and group 2. The point-biserial correlation coefficient is calculated as follows:

$$r_{pb} = \frac{M_1 - M_0}{s_n} \sqrt{\frac{n_1 n_0}{n^2}}$$

where s_n is the standard deviation used when data are available for every member of the population:

$$s_n = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2}$$

M_1 represents the mean value on the continuous variable X for all data points in group 1, and M_0 represents the mean value on the continuous variable X for all data points in group 2. Furthermore, n_1 is the number of data points in group 1, n_2 is the number of data points in group 2, and n is the total sample size.

Based on the formula, we obtain $r_{pb} = -0.7260335016671369$, indicating a strong correlation between the two variables within the range of 0.6-0.8 in this point-biserial correlation analysis, where $p_value = 3.649\ 991\ 855\ 983\ 487 \times 10^{-36}$. As shown in Figure 8, the x-axis represents the momentum score of the current point, where positive momentum indicates player 1's higher momentum, and negative momentum indicates player 2's higher momentum. We can observe that when the momentum score is positive, the next game's winner is concentrated around the value of 0, indicating a higher probability of player 1 winning. Similarly, the opposite holds true. This visualization provides a more intuitive understanding of the data, supporting the strong correlation between momentum scores and the outcome of the game.

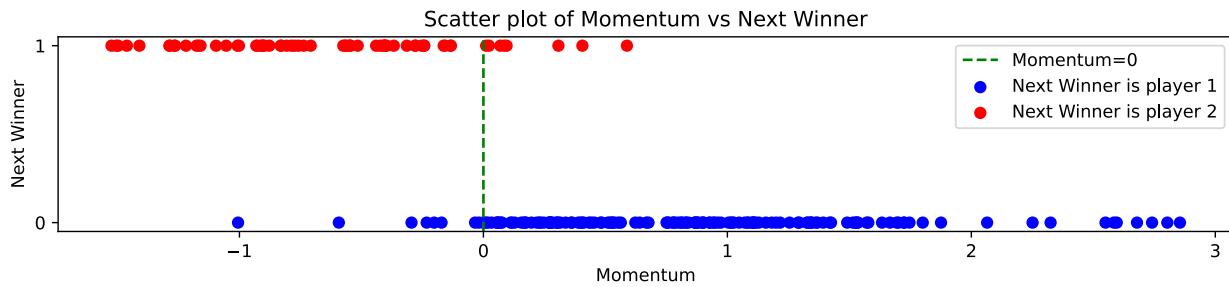


Figure 8: Relationship between Momentum Score and Next Game Winner

4.2 Randomness Testing

To test the coach's hypothesis that momentum is merely a random fluctuation, we can employ a stochastic simulation approach.

Calculation of actual momentum scores: Based on the momentum scoring formula defined earlier, we compute the momentum score for each scoring point in actual matches. We concatenate the distribution of momentum scores from the actual match data 100 times to align it with the randomly generated match results.

Simulation of random matches: In the simulation, the outcome of each point is completely random. Considering that the serving side has a slightly higher probability of winning, we can set the serving side's winning probability to be slightly higher than that of the receiving side. We then calculate the momentum scores under these random conditions and repeat the simulation 100 times to obtain the final distribution of momentum. We make the following assumptions:

(1) Null hypothesis (H_0): The momentum transitions in matches are random and do not differ significantly from the results of the random simulation.

(2) Alternative hypothesis (H_1): The momentum transitions in matches are non-random and differ significantly from the results of the random simulation.

Based on the random simulation, we performed a two-sample Kolmogorov-Smirnov (KS) test on matches 1301 and 1310 to determine whether the two samples are drawn from the same distribution. The simplified KS model yields a KS statistic that represents the maximum vertical distance between the empirical distribution functions of the two sample data sets. A larger KS statistic indicates a higher likelihood that the two samples are drawn from different distributions.

The p-value represents the probability of observing the current KS statistic or a more extreme value under the hypothesis that the two samples are drawn from the same distribution. If the p-value is smaller than a given significance level (e.g., 0.05), we can reject the hypothesis that the two samples are drawn from the same distribution.

For the random simulation of match 1301, we obtained a KS statistic of 0.309 and a p-value of 2.72876×10^{-12} . For match 1310, we obtained a KS statistic of 0.258 and a p-value of 2.56378×10^{-12} . Figure 9 displays the visual results of comparing the momentum from the random simulations with the actual momentum for these two matches. Combining the aforementioned decision rule, we can clearly reject the hypothesis that the two samples are drawn from the same distribution, thus providing evidence in support of the alternative hypothesis (H_1): the momentum transitions in matches are non-random and differ significantly from the results of the random simulation.

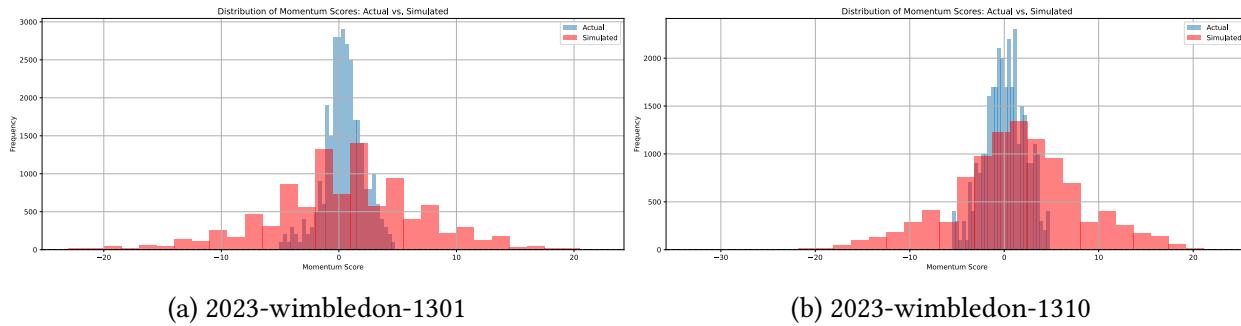


Figure 9: Distribution of Momentum Scores: Actual vs. Simulated

4.3 Machine Learning Pre-Experiment

To predict the changes in flow on the court, i.e., the shifts from favoring one player to favoring the other player, we define it as a swing.

We adopt the LSTM model to forecast swings using machine learning techniques.

Python programming language and the TensorFlow machine learning framework are used to implement the model, and an annotated dataset is employed for experimentation.

4.3.1 Preparation

- We select a preprocessed dataset containing time series data, where each time step corresponds to a momentum value. Initially, we manually annotate the match data by adding a new column labeled 'swing', indicating whether a swing occurred in the current game (1 if yes, otherwise 0).
- The data is read and normalized to scale the values between 0 and 1.
- The window size is defined as 20, determining the historical time steps for each sample.

4.3.2 Model Construction

- Use the Sequential model to create LSTM model.

- Add a LSTM layer as the first layer of the model.
- Add a dense layer with a sigmoid activation function as the output layer of the model to perform binary classification.
- Compile the model using the adam optimizer and binary crossentropy loss function.

4.3.3 Model Training

- The dataset is split into a training set (80%) and a test set (20%).
- The model is trained using the training set data, specifying the number of epochs (epochs=10) and batch size (batch_size=32).

4.3.4 Model Prediction

- The model is used to predict on the test set data.
- The predicted results are binary classified based on a threshold to determine if a change has occurred.

Since the dataset requires manual annotation and the test and validation sets are relatively small, we utilize visualization to present the results.

It can be seen from the graph that the accuracy of this method is relatively high. Under the condition that all results are 0, almost all of them have undergone a swing. This means that our model can predict the occurrence of swing.

4.4 Advice for Player

4.4.1 Overall Recommendations

Based on the analysis of the results, we can observe a strong correlation between the model we built for momentum and the actual swings. The results from the model can almost perfectly predict the occurrence of swings.

Based on the model results, we can provide the following recommendations to coaches:

- Focus on specific indicators of game turning points, such as break points, as these factors may indicate upcoming changes in the flow of the game.
- Train players to recognize and capitalize on turning points in the game, such as being mentally prepared and making tactical adjustments during continuous scoring or crucial moments. These adjustments may include modifications to serving strategies, receiving strategies, and running strategies.
- Analyze match data against different opponents to customize strategies for different game situations. In our model testing, we observed significant variations in specific turning point indicators for different players and matches.

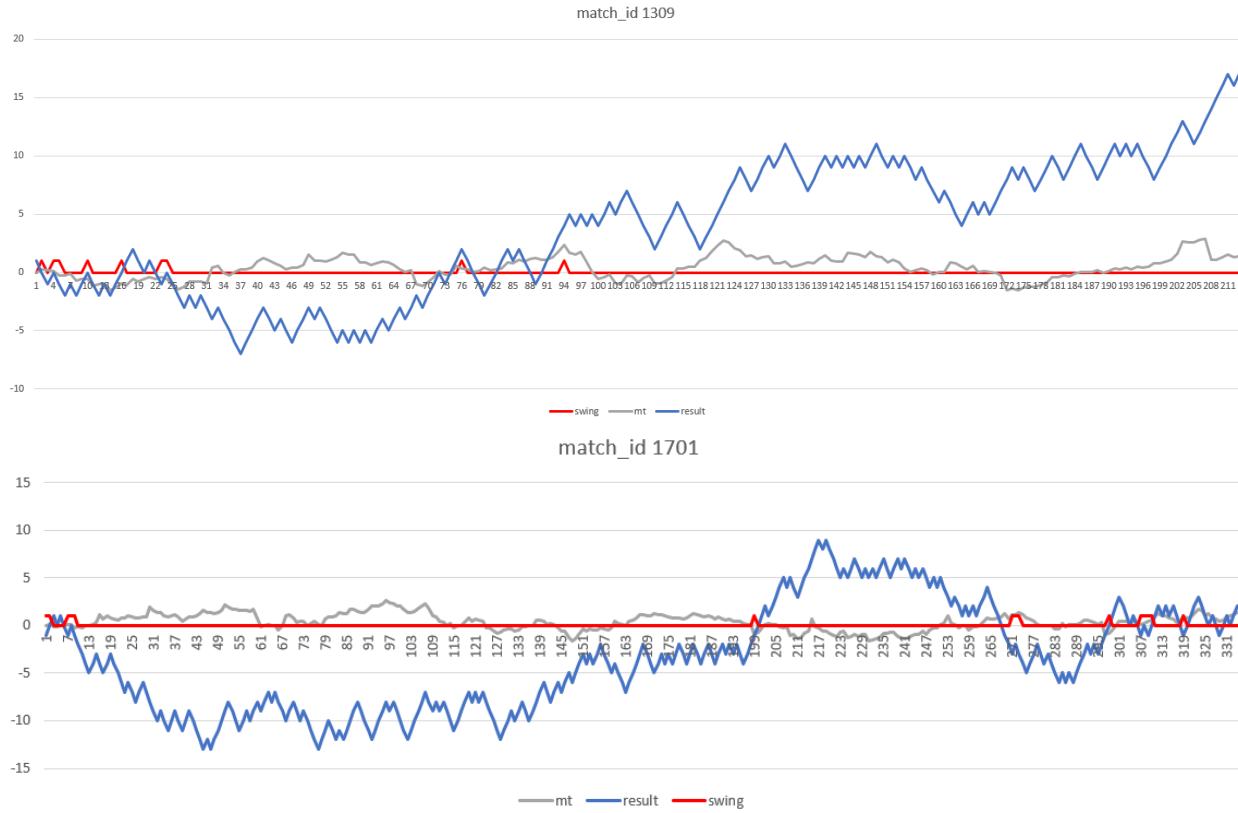


Figure 10: Prediction of Swing in game. Red line represents the swing in game if it is 1.

4.4.2 Specific Recommendations

Based on our parameter analysis, we provide the following recommendations for specific players:

From the analysis of Alcaraz's five matches, the characteristics of Alcaraz are as follows:

- Alcaraz's momentum is strongly correlated with his ability to score in the previous shot. He excels in continuous scoring and experiences a significant increase in momentum after scoring a point. This indicates that Alcaraz has strong fundamental skills. This parameter accounts for 0.8945 of the overall influence.
- The impact of Alcaraz's running distance difference on momentum is relatively small. There is no need to be overly concerned about the decrease in physical fitness due to longer running distances. This parameter accounts for 0.1440 of the overall influence.
- Winning break points can bring a strong momentum boost to Alcaraz. As long as he wins break points, he is likely to turn the tide of the game. In matches, he should make extra efforts to win break points. This parameter accounts for 0.6919 of the overall influence.
- The influence of his own errors and consecutive wins on his momentum is relatively small. This indicates that he has strong mental resilience.

From the analysis of Sinner's four matches, the characteristics of Sinner are as follows:

- Sinner's momentum is not strongly correlated with his ability to score in the previous shot.
- The impact of Sinner's running distance difference on momentum is significant. He can leverage opponent's running to gain an advantage. This parameter accounts for 0.7752 of the overall influence.
- Winning break points has a negative impact on Sinner's momentum. This may indicate a weaker mental resilience. After winning break points, he should maintain composure and continue to strive.
- Sinner is significantly affected by his own errors. He can work on reducing errors through training or maintaining composure after making mistakes.
- Consecutive wins by the opponent have a significant impact on Sinner. This parameter accounts for 0.3996 of the overall influence. He should strengthen his mental resilience and remain calm when facing consecutive wins by the opponent.

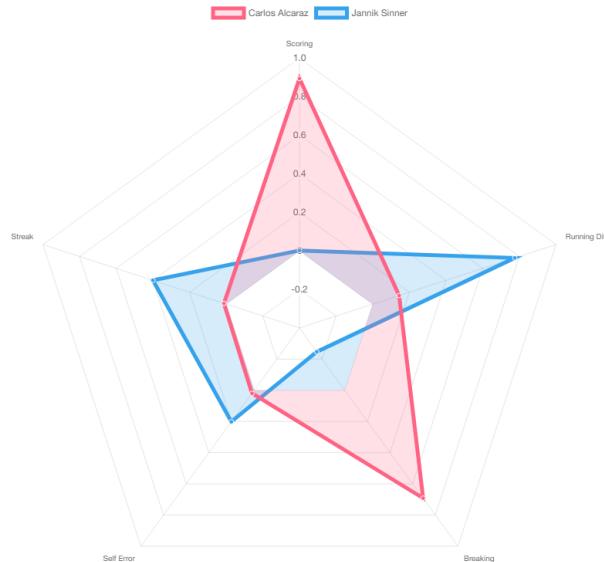


Figure 11: Radar chart of player Sinner and Alcaraz.

For other players, a similar analysis should be conducted based on their respective match data.

5 Experimental Analysis

5.1 Performance

We tested our model on the complete set of matches from the 2023 Wimbledon dataset, and the results are as follows:

Our model achieved an average accuracy of over 90% in predicting swings for almost all matches. Additionally, the average accuracy in predicting the winner of the next game reached over 60%. Please refer to Appendix A for the accuracy table. In the table, acc1 represents the accuracy of predicting the match flow, i.e., swings, while acc2 represents the accuracy of predicting the winner of the next game based on the current player's performance.

5.2 Generalization

We applied our model to two generalized datasets: the 2022 US Open competition and a badminton competition, specifically the 2022 BWF Tour.

Due to the need for manual labeling, we conducted limited testing on the badminton dataset. Additionally, since badminton has different rules from tennis, we were unable to calculate acc2.

6 Conclusions

Drawing upon statistical analysis and machine learning expertise, we have observed a robust correlation between the momentum model we constructed and the real-world swings in tennis matches. Notably, our model exhibits a nearly flawless ability to predict swing occurrences. In addition, after we quantify the indicator of momentum, players can not only evaluate their own status based on the indicator, but also measure the strength and competition preferences of the opponents they will encounter based on this indicator to better seize opportunities. Leveraging this valuable insight, we can offer coaches and players actionable recommendations for effectively managing and capitalizing on these pivotal moments in the game.

1. Understand the opponent:

- Study the opponent's style and habits: Utilize previous data and video analysis to understand the opponent's playing style, strengths, and weaknesses. Identify moments where the opponent is likely to gain momentum or potentially falter, allowing you to change the pace of the game during these moments.
- Prepare targeted strategies: Based on the opponent's performance fluctuations, prepare corresponding strategies. For example, if the opponent starts strongly, adopt a more defensive strategy, conserve energy, and wait for an opportunity when they may weaken.

2. Adjust self-mentality:

- Strengthen mental resilience: Cultivate mental resilience to cope with different situations during the match, maintaining composure even when facing unfavorable momentum.
- Set short-term goals: Setting small goals during the match can help maintain focus and motivation, such as aiming to win each service game or reducing unforced errors.

3. Control the pace of the match:

- Utilize momentum changes: Identify momentum shifts indicated by data analysis and recognize when to increase aggression or adopt a conservative approach. Increase aggression

when the opponent starts making more errors, and exercise patience when waiting for opportunities.

- Vary tactics: Flexibly changing tactics based on the game situation is a key strategy that can make it difficult for opponents to adapt. This includes changing the serving method, adjusting the baseline, and other aspects. Based on our model testing results, we observed significant changes in specific turning point indicators for different players and matches.
- Seizing the advantage of scoring with a broken serve can be a pivotal moment in a tennis match, as it not only allows the server to gain a valuable point but also disrupts the opponent's rhythm and confidence.

It is important to note that these recommendations are based on data analysis and the principles of sports psychology. Coaches should adapt and tailor these strategies to individual players and their specific game situations.

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Appendices

Appendix A Accuracy Table

2023 Wimbledon			2022 US Open			2022 BWF World Tour	
id	acc1	acc2	id	acc1	acc2	id	acc1
1301	90.733	0.637583893	1101	90.877	0.671328671	WS-11	84.387
1302	92.311	0.608040201	1102	94.566	0.606741573	WS-12	87.589
1303	91.762	0.681818182	1103	93.915	0.51010101	WS-21	84.913
1304	92.149	0.576119403	1104	90.137	0.727272727	MS-11	85.132
1305	93.841	0.602459016	1105	90.887	0.643678161	MS-12	87.803
1306	93.625	0.603030303	1106	89.393	0.631578947	MS-21	84.325
1307	91.853	0.547826087	1107	92.904	0.726027397		
1308	92.362	0.505319149	1108	93.71	0.672222222		
1309	90.352	0.701421801	1109	88.15	0.661354582		
1310	91.221	0.528481013	1110	92.263	0.577142857		
1311	94.686	0.68452381	1111	92.371	0.55511811		
1312	92.86	0.648351648	1112	88.128	0.558282209		
1313	93.091	0.559027778	1113	89.293	0.742268041		
1314	95.914	0.617486339	1114	91.315	0.602649007		
1315	90.397	0.571428571	1115	94.97	0.59602649		
1316	94.792	0.703030303	1116	89.432	0.615763547		
1401	93.806	0.614349776	1117	94.877	0.676855895		
1402	94.349	0.558823529	1118	93.007	0.618473896		
1403	94.322	0.791666667	1119	94.962	0.538888889		
1404	90.381	0.584507042	1120	94.982	0.687830688		
1405	89.098	0.680751174	1121	94.647	0.508108108		
1406	93.785	0.642487047	1122	93.935	0.582352941		
1407	93.389	0.632218845	1123	90.066	0.582524272		
1408	93.335	0.665427509	1124	90.065	0.583892617		
1501	90.384	0.64171123	1125	90.421	0.526132404		
1502	93.395	0.660649819	1126	92.089	0.635514019		
1503	92.354	0.638743455	1127	93.067	0.61038961		
1504	89.628	0.618604651	1128	88.354	0.643962848		
1601	92.517	0.617834395	1129	90.702	0.648854962		
1602	92.777	0.6	1130	90.243	0.552238806		
1701	91.32	0.602409639	1131	88.612	0.577844311		
Avg	92.477	0.623423622	Avg	91.688	0.615207091		

Report on Use of AI

1. OpenAI ChatGPT 4.0

Prompt1: Please convert this content into English based on your knowledge of mathematical modeling, machine learning, and statistics, and adjust it to conform to the standards of academic papers and the format of latex. If there is any missing or necessary information, please add it directly to the paragraph. Please check your work before replying. [Paragraph Here]

Output: Please note that the provided explanation is based on our mathematical modeling, machine learning, and statistical knowledge. We have transformed the content into English and adjusted it to conform to the standards of an academic paper and LaTeX format. If there are any omissions or necessary additions, please make the required adjustments before submitting your work. [Paragraph Here]

2. OpenAI ChatGPT 4.0

Query1: Please describe the operation process of the PSO algorithm.

Output: The PSO algorithm consists of the following steps:

- Initialization: Initialize a population of particles with random positions and velocities within the parameter space.
- Evaluation: Evaluate the fitness of each particle by applying the model with the corresponding parameter values and comparing the results with the actual momentum data.
- Update Personal Best: Update the personal best position and fitness for each particle based on its current fitness and position.
- Update Global Best: Update the global best position and fitness by selecting the particle with the highest fitness value among all particles.
- Velocity and Position Update: Update the velocities and positions of each particle based on its personal best, global best, and the inertia weight.
- Termination: Repeat steps 2-5 until a termination criterion is met (e.g., maximum number of iterations or desired fitness level reached).